MAC 2311 Recitation 2/26/21 Notes

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1 Problems

Problem 4: If a snowball melts so that its surface area decreases at a rate of 5 cm^2/min, find the rate at which the diameter decreases when the diameter is 11 cm.

Solution:

$$A = 4\pi r^2$$
$$\because D = 2r \quad \Rightarrow \frac{D}{2} = r$$
$$\Rightarrow A = 4\pi \left(\frac{D}{2}\right)^2 \quad \Rightarrow A = \frac{4\pi D^2}{4} \quad \Rightarrow A = \pi D^2$$
$$\Rightarrow \frac{dA}{dt} = 2\pi D \frac{dD}{dt} \quad \Rightarrow \frac{1}{2\pi D}\frac{dA}{dt} = \frac{dD}{dt}$$
$$\therefore \frac{dD}{dt} = \frac{5}{22\pi}\text{cm/in}$$

Because it is decreasing,

$$\therefore \frac{dn}{dt} = -\frac{5}{22\pi}\text{cm/min}$$

□

Problem 6: Two cars start moving from the same point. One travels south at 48mi/h and the other travels west at 20mi/h. At what rate is the distance between the cars increasing three hours later?

Solution:

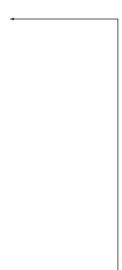

Figure 1:

$$v_1^2 + v_2^2 = v_b^2 \quad \Rightarrow v_b = \sqrt{v_1^2 + v_2^2} \quad \Rightarrow v_b = \sqrt{(48 \text{ mi/h})^2 + (20 \text{ mi/h})^2} \tag{1}$$

$$\Rightarrow v_b = \sqrt{2704} \quad \Rightarrow v_b = 52 \text{ mi/h} \tag{2}$$

Typo, correction (what is the distance 3 hrs later).

$$\because v_b = \frac{dx_b}{dt} \quad \Rightarrow \int_0^{x_b} dx_b = \int_0^3 v_b dt \quad \Rightarrow x_b = v_b t\Big]_0^3 \tag{3}$$

$$\Rightarrow x_b = v_b(3) \quad \Rightarrow x_b = (52\text{mi/h})(3\text{h}) \tag{4}$$

$$\therefore x_b = 156\text{mi} \tag{5}$$

□

Problem 10: Boyle's Law states that when a sample of gas is compressed at a constant temperature, the pressure P and volume V satisfy the equation $PV = C$, where C is a constant. Suppose that at a certain instant the volume is 600 cm^3, the pressure is 100kPa and the pressure is increasing at a rate of 20kPa/min. At what rate is the volume decreasing at this instant?

Solution:

$$P(t)V(t) = c \quad \Rightarrow P\frac{dV}{dt} + V\frac{dP}{dt} = 0 \tag{6}$$

$$\Rightarrow P\frac{dr}{dt} = -V\frac{dP}{dt} \quad \Rightarrow \frac{dV}{dt} = -\frac{V}{P}\frac{dP}{dt} \tag{7}$$

$$\therefore \frac{dV}{dt} = -120\frac{\text{cm}^3}{\text{min}} \tag{8}$$

□

HW 3.10-5: Find the linear approximation of the function

$$f(x) = \sqrt{4 - x}$$

at $a = 0$. Find $L(x) =$

and, use $L(x)$ to approximate the numbers $\sqrt{3.9}$ and $\sqrt{3.99}$. (Round your answers to four decimal places.)

Solution:

Taylor series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x)}{n!}(x - a)^n \tag{9}$$

In this exercise, expand to the first derivative, i.e.,

$$f(x) = \sum_{n=0}^{1} \frac{f^{(n)}(a)}{n!}(x - a)^n \tag{10}$$

So,

$$f(x) = f^{(0)}(a) + f^{(1)}(a)(x - a) \tag{11}$$

$$\Rightarrow f(x) = \sqrt{4 - a} - \frac{1}{2\sqrt{4 - a}}(x - a) \tag{12}$$

$\because a = 0$,

$$\Rightarrow f(x) = \sqrt{4} - \frac{1}{2\sqrt{4}}x \tag{13}$$

$$\therefore f(x) = 2 - \frac{x}{4} \tag{14}$$

To approximate the numbers $\sqrt{3.9}$ and $\sqrt{3.99}$,

$$\because f(x) = \sqrt{4-x} \quad \Rightarrow 4 - x = 3.9 \quad \Rightarrow x = 0.1 \tag{15}$$

So for (14),

$$f(0.1) = 2 - \frac{0.1}{4} = 1.9750 \tag{16}$$

For $\sqrt{3.99}$,

$$\because f(x) = \sqrt{4-x} \quad \Rightarrow 4 - x = 3.99 \quad \Rightarrow x = 0.01 \tag{17}$$

So from (14),

$$f(0.01) = 2 - \frac{0.01}{4} = 1.9975 \tag{18}$$

□

Problem 10: Use a linear approximation (or differentials) to estimate the given number.

$$\sqrt{100.4}$$

Solution: Utilize the Taylor series at $a = 100$ to the first order.

$$f(x) = \sum_{n=0}^{1} \frac{f^{(n)}(a)}{n!}(x-a)^n \tag{19}$$

So,

$$f(x) = f^{(0)}(a) + f^{(1)}(a)(x-a) \tag{20}$$

We choose $a = 100$ because we want the nearest square-able number.

$$\Rightarrow f(x) = f(100) + f^{'}(100)(x-100) \tag{21}$$

Next, choose $f(x) = \sqrt{x}$. Thus,

$$f(x) = \sqrt{100} + \frac{1}{2\sqrt{100}}(x-100) = 10 + \frac{1}{2(10)}(x-100) \tag{22}$$

$$\Rightarrow f(x) = 10 + \frac{x}{20} - 5 \tag{23}$$

$$\therefore f(x) \approx \sqrt{5} + \frac{x}{20} \tag{24}$$

Now let $x = 100.4$,

$$\Rightarrow f(100.4) = 5 + \frac{100.4}{20} = 10.02 \tag{25}$$

□