– Chain rule

$$(f \circ g)'(x) = (f' \circ g)(x)\, g'(x)$$

$f'(x) = \frac{df}{dx}$ $\quad \because (f \circ g)(x) = f(g(x))$

$$(f \circ g)'(x) = (f'(g(x)))\, g'(x)$$

Let,

$y = f(u)$ and $u = g(x)$

$f'(u) = \frac{dy}{du}$

$$\frac{dy}{dx} = (f'(g))\, u'$$

$$\frac{dy}{dx} = \frac{dy}{du} \quad \text{and} \quad u' = \frac{du}{dx}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}}$$

#1

$y = \sqrt{1+2x}$

Write the function in the form

$y = f(u)$ and $u = g(x)$

then find

$\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Let,

$u = 1+2x$ and $y = (u)^{\frac{1}{2}}$

then

$\frac{du}{dx} = 2$ and $\frac{dy}{du} = \frac{1}{2}(u)^{-\frac{1}{2}}$

$\frac{dy}{du} = 2$ and $\frac{du}{dx} = \frac{1}{2}(u)^{-\frac{1}{2}}$

$\frac{dy}{dx} = \left[\frac{dy}{du}\frac{du}{dx}\right] \Rightarrow \frac{dy}{dx} = (u)^{-\frac{1}{2}}$

$\therefore u = 1+2x \Rightarrow \frac{dy}{dx} = (1+2x)^{-\frac{1}{2}}$

$(f \circ g)'(x) = f'(g(x))\,g'(x)$

$y = \cos\left((\ln|x|)^{\frac{1}{2}}\right)$

$(f \circ g \circ h)'(x) =$ $\left[(f \circ g) \circ h\right)'(x) = (f \circ g)'(h(x))h'(x)$

$\rightarrow h = \ln|x|$

$g = (h)^{\frac{1}{2}}$

$\rightarrow f = \cos(g)$

$\frac{dy}{du} = 2$ and $\frac{du}{dx} = \frac{1}{2}(u)^{-\frac{1}{2}}$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \Rightarrow \frac{dy}{dx} = (u)^{-\frac{1}{2}}$

$\therefore u = 1+2x \Rightarrow \frac{dy}{dx} = (1+2x)^{-\frac{1}{2}}$

$(f \circ g)'(x) = f'(g(x))\,g'(x)$

$y = \cos\left((\ln|x|)^{\frac{1}{2}}\right)$

$(f \circ g \circ h)'(x) = \left[(f \circ g) \circ h\right)'(x) = (f \circ g)'(h(x))h'(x)$

$\rightarrow h = \ln|x|$

$\rightarrow g = (h)^{\frac{1}{2}}$

$\rightarrow f = \cos(g)$

$$(f \circ g \circ h)'(x) = [(f \circ g) \circ h]'(x)$$

$$\Rightarrow (f \circ g)'[h(x)]\, h'(x)$$

$$\because (f \circ g)'(h(x)) = f'(g[h(x)])\, g'(h(x))$$

$$\therefore (f \circ g \circ h)'(x) = f'[g(h(x))]\, g'(h(x))\, h'(x)$$

x	f(x)	g(x)	f'(x)	g'(x)
1	3	8	3	6
2	4	1	5	2
3	7	2	6	1

$h(x) = f(g(x))$

find $h'(3)$

$(f \circ g)'(x) = f'(g(x))\,g'(x)$

$(f \circ g)(x) = f(g(x)) = h(x)$

$\Rightarrow h'(x) = f'(g(x))\,g'(x)$

$\Rightarrow h'(3) = f'(g(3))\,g'(3)$

$\because g(3) = 2$

$\Rightarrow h'(3) = f'(2)\,g'(3)$

$h'(3) = 5 \cdot 1 = 5$

implicit form and explicit form

$x^2 + y^2 = r^2$

where r = radius

implicit form

also,

$y^2 = r^2 - x^2 \Rightarrow y = \sqrt{r^2 - x^2}$

explicit form

$\frac{dy}{dx}$

For $y = \sin^{-1}(x)$

find $\frac{dy}{dx}$

$$\frac{dy(x)^2}{dx} = \frac{dy^2}{dx}$$

$\therefore y(x) = \sin^{-1}(x)$

$$\Rightarrow \boxed{\sin(y(x)) = x}$$

$$\frac{d}{dx}\left[\sin(y(x))\right] = \frac{d}{dx}[x]$$

$$\cos(y(x))\frac{dy(x)}{dx} = 1$$

$$\frac{dy(x)}{dx} = \frac{1}{\cos(y(x))} \Rightarrow \frac{dy(x)}{dx} = \frac{1}{\sqrt{1-\sin^2(y(x))}}$$

$\therefore \cos^2(y(x)) + \sin^2(y(x)) = 1$

$\Rightarrow \cos^2(y(x)) = 1 - \sin^2(y(x))$

$\Rightarrow \cos(y(x)) = \sqrt{1-\sin^2(y(x))}$

$$\sin(y(x)) = x$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$V = 2\pi r^2 h$$

and the volume and radius are changing with respect to time t

$$V(t) = 2\pi [r(t)]^2 h$$

$$\frac{dV(t)}{dt} = 4\pi r(t) \frac{dr}{dt} h$$

Find the slope of the curve

$y = 1 - 3x^2$

at a given point $P(x_1, y_1) = P(3, -25)$

and the eqn. of the tangent line @ P

$$\left.\frac{dy}{dx}\right|_{x=x_1} = -6x = -6(3) = -1$$

$$m = \left.\frac{dy}{dx}\right|_{x=x_1}$$

$\therefore m = -18$

$\therefore m = \frac{y - y_1}{x - x_1} \Rightarrow m(x - x_1) = y - y_1$

$\Rightarrow m(x - x_1) + y_1 = y$

$y = m(x - x_1) + y_1$

$P_1(3, -25)$

$m = -18$

~~$y = -18(x - 3) - 25$~~

~~$y = -18x + 36 - 25$~~

$y = -18(x - 3) - 25$

~~$\therefore y = -18x +$~~