# MAC 2311 Recitation 2

Anthony Asilador

January 21, 2021

## 1 Problems

**Definition:** Let $A \subseteq \mathbb{R}$, and let $c$ be a cluster point of $A$. For a function $f : A \to \mathbb{R}$, a real number $L$ is said to be a limit of $f$ at $c$ if, given any $\varepsilon > 0$, there exists a $\delta > 0$ such that if $x \in A$ and $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.

**Remark:** If the limit of $f$ at $c$ does not exist, we say that $f$ diverges at $c$.

**Theorem:** If $f : A \to \mathbb{R}$ and if $c$ is a cluster point of $A$, then $f$ can have only one limit at $c$.

**Squeeze Theorem:** Let $A \subseteq \mathbb{R}$, let $f, g, h : A \to \mathbb{R}$, and let $c \in \mathbb{R}$ be a cluster point of $A$. If

$$f(x) \leq g(x) \leq h(x) \quad \text{for all} \quad x \in A, x \neq c \tag{1}$$

and if $\lim\limits_{x \to c} f = L = \lim\limits_{x \to c} h$, then $\lim\limits_{x \to c} g = L$.

**Definition:** Let $I \subseteq \mathbb{R}$ be an interval, let $f : I \to \mathbb{R}$, and let $c \in I$. We say that a real number $L$ is the derivative of $f$ at $c$ if given any $\varepsilon > 0$ there exists $\delta(\varepsilon) > 0$ such that if $x \in I$ satisfies $0 < |x - c| < \delta(\varepsilon)$, then

$$\left| \frac{f(x) - f(c)}{x - c} - L \right| < \varepsilon \tag{2}$$

In this case we say that $f$ is differentiable at $c$, and we write $f'(c)$ for $L$. In other words, the derivative of $f$ at $c$ is given by the limit

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c} \tag{3}$$

provided this limit exists. (We allow the possibility that $c$ may be the endpoint of the interval.)

**Definition:** The derivative of the function $f(x)$, written $f'(x)$, is defined as

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \tag{4}$$

if the limit exists. If $y = f(x)$, then $\dfrac{dy}{dx} = f'(x)$. The $n^{\text{th}}$ derivative is

$$y^{(n)} = \frac{dy^{(n-1)}}{dx} = \frac{d}{dx}\left(\frac{d^{n-1}y}{dx^{n-1}}\right) = \frac{d^n y}{dx^n} = f^{(n)}(x) \tag{5}$$

## 2 Practice Problems

**Problem 1:**

$$\lim_{x\to\infty} \frac{x}{x-1} \quad (x \neq 1) \tag{6}$$

**Problem 2:**

$$\lim_{x\to\infty} (\sqrt{x+1})/x \quad (x > 0) \tag{7}$$

**Problem 3:**

$$\lim_{x\to\infty} \frac{\sqrt{x}-5}{\sqrt{x}+3} \quad (x > 0) \tag{8}$$

**Problem 4:**

$$\lim_{x\to 0} x\cos(1/x) = 0 \tag{9}$$

**Problem 5:** Let,

$$y = y_0 + v_0 t - \frac{1}{2} g t^2 \tag{10}$$

Use the definition of the derivative to find the instantaneous velocity.