# Homework 6 MAP 6642 Financial Mathematics II

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March 13, 2020

## 1 HW 6 Problems

Problem 1. Prove the following statement:

The NA price of a caplet with payoff

$$\delta \cdot (L(T;T,T+\delta) - \kappa)^+ \tag{1}$$

at time $T+\delta$ equals the NA price of a put option with the payoff

$$(1+\delta\cdot\kappa)\cdot\left(\frac{1}{1+\delta\cdot\kappa} - p(T,T+\delta)\right)^+ \tag{2}$$

at time $T$.

Solution:

$$\because L(t;S,T) = \frac{p(t,S)-p(t,T)}{(T-S)p(t,T)} \tag{3}$$

$$\Rightarrow L(T;T,T+\delta) = \frac{p(T,T)-p(T,T+\delta)}{(T+\delta-T)p(T,T+\delta)} \tag{4}$$

$$\therefore L(T;T,T+\delta) = \frac{1-p(T,T+\delta)}{\delta p(T,T+\delta)} \tag{5}$$

Thus,

$$\delta(L(T;T,T+\delta)-\kappa)^+ = \delta\left(\frac{1-p(T,T+\delta)}{\delta p(T,T+\delta)} - \kappa\right)^+ = \delta\left(\frac{1-p(T,T+\delta)-\kappa\delta p(T,T+\delta)}{\delta p(T,T+\delta)}\right)^+ \tag{6}$$

$$\Rightarrow \delta(L(T;T,T+\delta)-\kappa)^+ = \left(\frac{1-p(T,T+\delta)-\kappa\delta p(T,T+\delta)}{p(T,T+\delta)}\right)^+ = \left(\frac{1-p(T,T+\delta)[1+\kappa\delta]}{p(T,T+\delta)}\right)^+ \tag{7}$$

$$\Rightarrow \delta(L(T;T,T+\delta)-\kappa)^+ = \left(\frac{1}{p(T,T+\delta)} - [1+\kappa\delta]\right)^+ \tag{8}$$

$$\therefore \delta(L(T;T,T+\delta)-\kappa)^+ = \frac{(1+\kappa\delta)}{p(T,T+\delta)}\left(\frac{1}{(1+\kappa\delta)} - p(T,T+\delta)\right)^+ \tag{9}$$

At time $T+\delta$.

At time $T$,

$$\delta(L(T;T,T+\delta)-\kappa)^+ = p(T,T+\delta)\frac{(1+\kappa\delta)}{p(T,T+\delta)}\left(\frac{1}{(1+\kappa\delta)} - p(T,T+\delta)\right)^+ \tag{10}$$

$$\therefore \delta(L(T;T,T+\delta)-\kappa)^+ = (1+\kappa\delta)\left(\frac{1}{(1+\kappa\delta)} - p(T,T+\delta)\right)^+ \tag{11}$$

□